convinced him that the colour-changes which these reptiles undergo with such rapidity are not, as often believed, in harmony with their surroundings, but are regulated chiefly by light, temperature, excitement, fright, or health. We here reproduce a partial list of these experiments on the common chameleon :—

Specimen A. Placed in the sunlight so that but one side of the lizard was exposed to the rays.

Specimen B. Placed in the sunlight at an angle to entirely suffuse the reptile with the rays.

Specimen C. Placed in a dark box; temperature, 73° F.

Specimen D. Placed in a dark box; temperature, 50° F.

After fifteen minutes, the following results were noted :—

Specimen A. Was a dark brown on the side that had been exposed to the sun; the shadowed side was a pale brown, mottled with green.

Specimen B. A uniform brown, deeper than the dark side of specimen A.

Specimen C. When the cover of the box was drawn the lizard emerged in a brilliant coat of green.

Specimen D. Crawled sluggishly from the cold quarters. Its colour was a uniform slaty-grey.

One curious effect of sunlight and shadow was noticed. A specimen had been basking under a coarse wire grating. Becoming frightened at the approach of the observer, it changed its position. On the dark brown body was what had been the shadow of the grating, brilliantly imprinted in pale yellow. Within half a minute this pattern had entirely faded.

The book is copiously illustrated with reproductions of photographs taken by the author from living specimens, and most of them are of high excellence. In some cases, however, the reduction is too great, such figures as those of the European lizards and the glass-snakes and slow-worm (plates xxxi*a* and xxxvii.) being, from this cause, practically useless. The snake figured on plate lxxvii as *Cerastes vipera*, and stated to measure about two and a half feet, is a hornless *Cerastes cornutus*. The author appears to be unaware of the existence of such hornless specimens, otherwise he would not have written (p. 328) that it is "impossible to mistake the horned viper," and that *C. vipera* is, but for the absence of horns, much like *C. cornutus*. A three-colour process figure of the rhinoceros viper, "the most beautifully coloured of all poisonous snakes," is given as a frontispiece.

G. A. B.

THE CALCULUS OF VARIATIONS.

Leçons sur le Calcul des Variations. By Prof. J. Hadamard. Tome premier. Pp. viii+520. (Paris: A. Hermann et Fils, 1910.) Price 18 francs.

NO one could be more competent than M. Hadamard to deal with the calculus of variations, and when this work is completed it will be a most valuable exposition of the present state of the subject. It is significant that in the first lines of his preface the author expresses the view that the calculus of variations is only a first chapter of the functional calculus (*calcul fonctionnel*) of Volterra, Pincherle, &c., and he gives, in fact, a short chapter on this new theory (pp. 281–312). But the analysis, in this volume, is mostly of a more familiar kind.

In fact, the first step in any actual case that naturally presents itself is still the classical one of Lagrange, by which we obtain a differential equation, or a set of differential equations. For simplicity, suppose the varied integral to be $\int f(x, y, y')dx$, then the differential equation is of the second order, and its solution is said to form a family of extremals. Supposing that the limiting values of x and the corresponding values of y to be given, then in the general case we may expect to find one extremal satisfying the terminal conditions. But it by no means follows that this curve really makes the given integral a maximum or minimum; an example due to Scheeffer is given on p. 45, which brings out the point very clearly. In this case the extremal found from the differential equation is $y=0$, and the corresponding value of the integral is 0; nevertheless, analytical curves can be drawn, as close as we please to $y=0$, which make the integral negative.

In any case, a solution obtained from an extremal is only a relative one; that is, the extremal gives a maximum or minimum value of the integral relatively to adjacent paths. And here it is important to define what we mean by *adjacent*, a fact first fully realised by Weierstrass, whose definition of adjacency of the pth order is given on p. 49. We may have, for instance, two curves each passing through the terminal points A, B, and as close together as we please, but one may be of continuous, the other of discontinuous curvature. Now, if we have a varied integral involving higher differential coefficients than y', we must exclude curves of discontinuous curvature, otherwise the problem becomes meaningless, and similarly in other cases.

After the limitations of the problem have thus been touched upon, book ii. deals with the first variation, and the conditions of the first order, including variable limits. Among other interesting points we have Weierstrass's transformation to homogeneous coordinates, a discussion of foci (points on the envelope of a family of extremals), and two very useful innovations due to M. Hadamard. If $\int f(x, y, y')dx$ is the varied integral, the *figurative* is defined to be the curve $f(x, y, y')=u$, in which u, y' are regarded as current coordinates, and x, y as constants. The *figuratrix* is defined as the polar reciprocal of the figurative with respect to $x^2+y^2=1$. By means of these curves the author is able to put various analytical conditions into a vivid geometrical shape. It may be added that book ii. contains the discussion of various classical problems, such as brachistochrones, least action, the Hamiltonian equations of dynamics, &c.

Book iii. introduces the second variation, and goes more deeply into the methods of Weierstrass, as well as those of Jacobi, Clebsch, Hilbert, Kneser, and others. We arrive ultimately at a statement, in various forms, of sufficient conditions for a minimum (pp. 389, 397), deduced mainly from the properties of a pencil of extremals, and a brief discussion of the necessary conditions, illustrated by examples (chapter iii.). The remaining chapters deal with variable limits, discontinuous solutions, Osgood's theorem in

connection with the strict minimum, and various other topics. Finally, there is a note on implicit functions.

Various interesting special theorems occur, by the way; as an instance, we have the theorem that if y vanishes for $x=a$ and $x=b$, the integral

$$\int_a^b \{(a-b)^2\left(\frac{dy}{dx}\right)^2 - \pi^2 y^2\} dx$$

is never negative.

It will be seen that this treatise is more for the advanced student than for the beginner; in fact, as the author expressly takes the theory of the differential and integral calculus for granted, the reader should be prepared with a good knowledge of analysis, including function-theory. In any case, the subject is intrinsically difficult, owing to the vagueness of the data when the problem is put in its general form; it is rather a matter of surprise that so much has been done, without unduly restricting the nature of the functions involved.

In conclusion, it should be stated that the treatise is based upon a course of lectures at the Collège de France, and that the *redaction* has been carried out by M. Fréchet, to whom M. Hadamard makes his acknowledgments G. B. M.

HYDROELECTRIC ENGINEERING.

Hydroelectric Developments and Engineering. A Practical and Theoretical Treatise on the Development, Design, Construction, Equipment, and Operation of Hydroelectric Transmission Plants. By F. Koester. Pp. xxv+454. (New York: D. van Nostrand Company; London: A. Constable and Co., Ltd., 1909.) Price 21*s.* net.

HYDROELECTRIC power plants do not call for the same attention in this country as in America and on the European continent. Yet what English engineer who has visited such installations has not a store of vivid recollections and happy experiences? The mountains and the forests, the streams and the waterfalls—for the generating stations of hydroelectric plants are usually away out among the beauties of nature—all bring back memories of pleasant tours and the like, whilst so far from destroying the attractiveness of their surroundings by harnessing nature's forces in this way, the author of the present work maintains that the scenery has at times been made more interesting, when proper attention has been paid to the architecture and situation of the buildings. This opinion is well upheld by many of the splendid photographs reproduced so well in this large volume.

The title of the book, however, is certainly ambitious, and, criticised from this point of view, we fear that the treatment on the whole is too general and descriptive, even to the point at times of being meagre, to be of great service to those directly connected with hydro-power plant installations. This will be further understood from the table of contents, which comprises chapters on dams, headrace, penstocks, power plant, mechanical equipment, electrical equipment, electrical transmission, substations, line protection, and a long list of developments, any one of which could occupy such a volume by itself. Hence it is almost inevitable that only a bird's-eye view could be given when all these subjects were brought within the compass of one book. It may be recalled that this popular mode of treatment appears to meet with more favour in America than in countries this side of the Atlantic. With this one reservation, however, we have nothing but praise for the general excellence of the book, the care devoted to its arrangement, and the high quality of its illustrations.

To show that the writer is well up-to-date, it is only necessary to refer to a few of the new features in hydroelectric developments which are dealt with in their respective chapters:—Airshafts and equalising chambers in connection with pressure tunnels; seamless welded, flangeless, telescoping penstocks to facilitate shipment and to eliminate expansion joints; siphon system, in contradistinction to the inverted siphon; impulse wheels with draft tubes and multiple, non-water-wasting nozzles; compound turbine on a single shaft, the discharge of one being the supply of the other; rapid and complete turbine tests by certain methods and autographic recording device; 30,000-volt generators and their efficient protective devices against lightning. Unique combination of single and three-phase high-tension transmission systems from three-phase generators; wagon-panel switchboard systems; segregation and decentralisation of switchboards; continuous water-flow grounders and horngaps with micrometric setting. Two-legged transmission towers and line-crossing protection.

At the end of each chapter is appended a bibliography of works and papers to which the student may turn for further information; this compilation is by no means the least valuable feature of the book.

Occasionally the author's treatment includes matter where his judgment seems to have been less sound. Thus in discussing electric generators, he states there are three types—the inductor, the revolving armature, and the revolving field. But surely there is no reason for treating all these at equal length?—indeed, little harm would have been done if the discussion of the first two types had been omitted completely in describing modern high-tension machines, unless, of course, the author intended to enter into the province of the designer in order to bring out certain advantages in the older types which have recently become prominent. Nor is sufficient attention paid to the development of high-speed water-turbine sets of large output. It would have been well to have supplied a table giving outputs and speeds of modern turbine sets for the various classes of turbines.

Here and there an error has been allowed to remain in the text, whilst at times important questions, such as the effect of capacity in transmission lines, have been omitted.

Following a very useful and well-written chapter on line protection (lightning arresters), the last section of the book is devoted to a detailed description of eight modern American and European hydroelectric developments, which serve well to show the immense advance made in water-power installations during recent years.

STANLEY P. SMITH.